

# Adding defects to the $4d \mathcal{N} = 2$ superconformal bootstrap

Madalena Lemos

CERN

Pre-Strings 2019  
Jul 5 2019, Leuven

together with L. Bianchi and M. Meineri

# Outline

## ① The bootstrap program

Adding defects

Universality in defect CFT

## ② Defects in $4d$ $\mathcal{N} = 2$ SCFTs

## ③ Summary & Outlook

# Outline

## 1 The bootstrap program

Adding defects

Universality in defect CFT

## 2 Defects in $4d \mathcal{N} = 2$ SCFTs

## 3 Summary & Outlook

# Conformal Bootstrap

## Conformal field theory defined by

Set of local operators and *all* their correlation functions

# Conformal Bootstrap

## Conformal field theory defined by

Set of local operators and *all* their correlation functions

## Operator Product Expansion

$$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_{k\text{prim.}} f_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_k} x^{\Delta_k - \Delta_1 - \Delta_2} (\mathcal{O}_k(0) + \dots)$$

→ Finite radius of convergence

# Conformal Bootstrap

## Conformal field theory defined by

Set of local operators and *all* their correlation functions

## Operator Product Expansion

$$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_{k\text{prim.}} f_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_k} x^{\Delta_k - \Delta_1 - \Delta_2} (\mathcal{O}_k(0) + \dots)$$

- Finite radius of convergence
- $n$ -point function by recursive use of the OPE until  $\langle \mathbb{1} \rangle = 1$

# Conformal Bootstrap

Conformal field theory defined by

$$\{\mathcal{O}_{\Delta,\ell,\dots}(x)\} \text{ and } \{f_{\mathcal{O}_i\mathcal{O}_j\mathcal{O}_k}\}$$

Operator Product Expansion

$$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_{k\text{prim.}} f_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_k} x^{\Delta_k - \Delta_1 - \Delta_2} (\mathcal{O}_k(0) + \dots)$$

- Finite radius of convergence
- $n$ -point function by recursive use of the OPE until  $\langle \mathbb{1} \rangle = 1$

# Conformal Bootstrap

Conformal field theory defined by

$$\{\mathcal{O}_{\Delta,\ell,\dots}(x)\} \text{ and } \{f_{\mathcal{O}_i\mathcal{O}_j\mathcal{O}_k}\}$$

Operator Product Expansion

$$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_{k\text{prim.}} f_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_k} x^{\Delta_k - \Delta_1 - \Delta_2} (\mathcal{O}_k(0) + \dots)$$

- Finite radius of convergence
- $n$ -point function by recursive use of the OPE until  $\langle \mathbb{1} \rangle = 1$

Subject to

- ▶ Unitarity



# Conformal Bootstrap

Conformal field theory defined by

$$\{\mathcal{O}_{\Delta,\ell,\dots}(x)\} \text{ and } \{f_{\mathcal{O}_i\mathcal{O}_j\mathcal{O}_k}\}$$

Operator Product Expansion

$$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_{k\text{prim.}} f_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_k} x^{\Delta_k - \Delta_1 - \Delta_2} (\mathcal{O}_k(0) + \dots)$$

- Finite radius of convergence
- $n$ -point function by recursive use of the OPE until  $\langle \mathbb{1} \rangle = 1$

Subject to

- ▶ Unitarity
- ▶ Associativity of the operator product algebra

# Conformal Bootstrap

Conformal field theory defined by

$$\{\mathcal{O}_{\Delta,\ell,\dots}(x)\} \text{ and } \{f_{\mathcal{O}_i\mathcal{O}_j\mathcal{O}_k}\}$$

Operator Product Expansion

$$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_{k\text{prim.}} f_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_k} x^{\Delta_k - \Delta_1 - \Delta_2} (\mathcal{O}_k(0) + \dots)$$

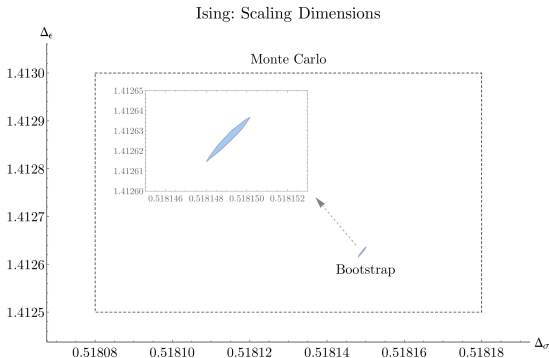
- Finite radius of convergence
- $n$ -point function by recursive use of the OPE until  $\langle \mathbb{1} \rangle = 1$

Subject to

- ▶ Unitarity
- ▶ Associativity of the operator product algebra
- ⇔ Crossing equations for *all* four-point functions

# 3d Ising Model

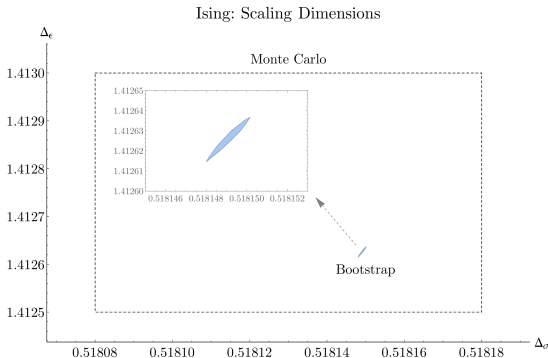
[Poland Simmons-Duffin Kos, Simmons-Duffin, Poland Simmons-Duffin Kos Vichi]



One  $\mathbb{Z}_2$ -even, one  $\mathbb{Z}_2$ -odd relevant scalar operator

# 3d Ising Model

[Poland Simmons-Duffin Kos, Simmons-Duffin, Poland Simmons-Duffin Kos Vichi]

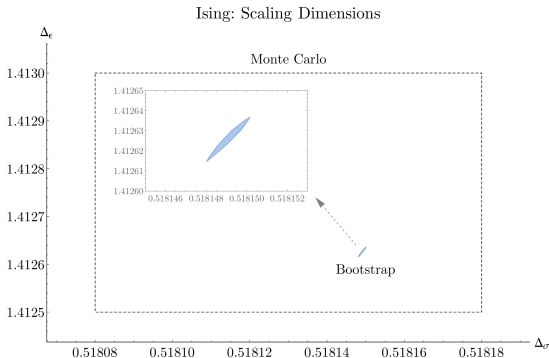


One  $\mathbb{Z}_2$ -even, one  $\mathbb{Z}_2$ -odd relevant scalar operator

► Lots of progress in delineating the space of theories

# 3d Ising Model

[Poland Simmons-Duffin Kos, Simmons-Duffin, Poland Simmons-Duffin Kos Vichi]



One  $\mathbb{Z}_2$ -even, one  $\mathbb{Z}_2$ -odd relevant scalar operator

- ▶ Lots of progress in delineating the space of theories
- ▶ and in bootstrapping specific CFTs [see Pufu's talk]

# Outline

## 1 The bootstrap program

Adding defects

Universality in defect CFT

## 2 Defects in $4d \mathcal{N} = 2$ SCFTs

## 3 Summary & Outlook

# Conformal Bootstrap

Local operators *do not* uniquely define a CFT

# Conformal Bootstrap

Local operators *do not* uniquely define a CFT

→ theories can have same local operators



# Conformal Bootstrap

## Local operators *do not* uniquely define a CFT

- theories can have same local operators
- and different spectrum of extended operators

# Conformal Bootstrap

## Local operators *do not* uniquely define a CFT

- theories can have same local operators
- and different spectrum of extended operators
  - ↪ line, surfaces, ...

# Conformal Bootstrap

## Local operators *do not* uniquely define a CFT

- theories can have same local operators
- and different spectrum of extended operators
  - ↪ line, surfaces, ...
- e.g., local operators do not distinguish between  $SU(2)$  and  $SO(3)$  gauge groups

# Conformal Bootstrap

## Local operators *do not* uniquely define a CFT

- theories can have same local operators
- and different spectrum of extended operators
  - ↪ line, surfaces, ...
- e.g., local operators do not distinguish between  $SU(2)$  and  $SO(3)$  gauge groups
- consistent to restrict to local operators

# Conformal Bootstrap

## Local operators *do not* uniquely define a CFT

- theories can have same local operators
- and different spectrum of extended operators
  - ↪ line, surfaces, ...
- e.g., local operators do not distinguish between  $SU(2)$  and  $SO(3)$  gauge groups
- consistent to restrict to local operators
- but want to move beyond those!

# Conformal Bootstrap

## Local operators *do not* uniquely define a CFT

- theories can have same local operators
- and different spectrum of extended operators
  - ↪ line, surfaces, ...
- e.g., local operators do not distinguish between  $SU(2)$  and  $SO(3)$  gauge groups
- consistent to restrict to local operators
- but want to move beyond those!

## Conformal defects

# Conformal Bootstrap

## Local operators *do not* uniquely define a CFT

- theories can have same local operators
- and different spectrum of extended operators
  - ↪ line, surfaces, ...
- e.g., local operators do not distinguish between  $SU(2)$  and  $SO(3)$  gauge groups
- consistent to restrict to local operators
- but want to move beyond those!

## Conformal defects

- Break as little symmetry as possible

# Conformal Bootstrap

## Local operators *do not* uniquely define a CFT

- theories can have same local operators
- and different spectrum of extended operators
  - ↪ line, surfaces, ...
- e.g., local operators do not distinguish between  $SU(2)$  and  $SO(3)$  gauge groups
- consistent to restrict to local operators
- but want to move beyond those!

## Conformal defects

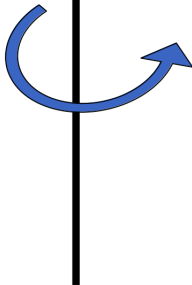
- Break as little symmetry as possible
- Preserve full conformal algebra on defect



# Defects in conformal field theories

$d$ - dimensions

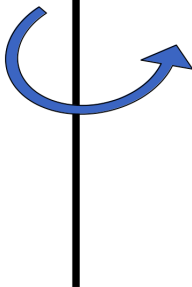
$\rightsquigarrow$   $p$ -dimensional defect



# Defects in conformal field theories

$d$ - dimensions

$\rightsquigarrow$   $p$ -dimensional defect



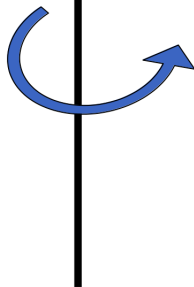
$d$ - dimensional conformal symmetry

$$so(d + 1, 1)$$

# Defects in conformal field theories

$d$ - dimensions

$\rightsquigarrow$   $p$ -dimensional defect



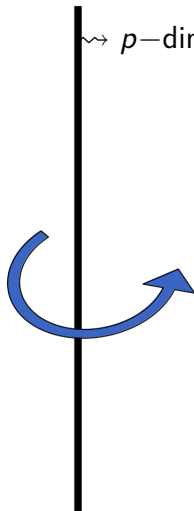
$d$ - dimensional conformal symmetry

$so(d + 1, 1)$

$\downarrow$  broken to

# Defects in conformal field theories

$d$ - dimensions



$\rightsquigarrow$   $p$ -dimensional defect

$d$ - dimensional conformal symmetry

$$so(d + 1, 1)$$

$\downarrow$  broken to

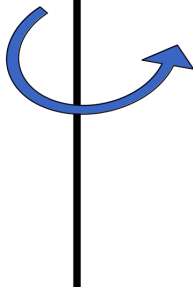
$$\underbrace{so(p + 1, 1)}$$

conformal symmetry on defect

# Defects in conformal field theories

$d$ - dimensions

$\rightsquigarrow$   $p$ -dimensional defect



$d$ - dimensional conformal symmetry

$$so(d + 1, 1)$$

$\downarrow$  broken to

$$so(d - p) \oplus \underbrace{so(p + 1, 1)}$$

conformal symmetry on defect

# Defects in conformal field theories

$d$ - dimensions

$\rightsquigarrow$   $p$ -dimensional defect

$d$ - dimensional conformal symmetry

$$so(d + 1, 1)$$

$\downarrow$  broken to

$$\rightsquigarrow so(d - p) \oplus \underbrace{so(p + 1, 1)}$$

rotations around defect

conformal symmetry on defect



# Defects in conformal field theories

## On defect

→ All said about CFTs applies

# Defects in conformal field theories

## On defect

- All said about CFTs applies
  - ! except: no stress tensor



# Defects in conformal field theories

## On defect

- All said about CFTs applies
  - ! except: no stress tensor
- OPE, crossing symmetry, ...

# Defects in conformal field theories

## On defect

- All said about CFTs applies
  - ! except: no stress tensor
- OPE, crossing symmetry, ...

## In bulk

- Still have OPE

# Defects in conformal field theories

## On defect

- All said about CFTs applies
  - ! except: no stress tensor
- OPE, crossing symmetry, ...

## In bulk

- Still have OPE
- Not enough to compute correlators:  $\langle \mathcal{O} \rangle \propto \frac{a_{\mathcal{O}}}{|x_{\perp}|^{\Delta_{\mathcal{O}}}}$

# Defects in conformal field theories

## On defect

- All said about CFTs applies
  - ! except: no stress tensor
- OPE, crossing symmetry, ...

## In bulk

- Still have OPE
- Not enough to compute correlators:  $\langle \mathcal{O} \rangle \propto \frac{a_{\mathcal{O}}}{|x_{\perp}|^{\Delta_{\mathcal{O}}}}$

## Bulk to defect OPE

$$\rightarrow \underbrace{\mathcal{O}}_{\text{bulk operator}} \sim \sum_i b_i x_{\perp}^{\hat{\Delta}-\Delta} \left( \underbrace{\hat{\mathcal{O}}_i}_{\text{defect operator}} + \dots \right)$$

# Defects in conformal field theories

## On defect

- All said about CFTs applies
  - ! except: no stress tensor
- OPE, crossing symmetry, ...

## In bulk

- Still have OPE
- Not enough to compute correlators:  $\langle \mathcal{O} \rangle \propto \frac{a_{\mathcal{O}}}{|x_{\perp}|^{\Delta_{\mathcal{O}}}}$

## Bulk to defect OPE

$$\rightarrow \underbrace{\mathcal{O}}_{\text{bulk operator}} \sim \sum_i b_i x_{\perp}^{\hat{\Delta}-\Delta} \underbrace{(\hat{\mathcal{O}}_i + \dots)}_{\text{defect operator}}$$

Subject to associativity of operator product algebra

# Defect operators

## Breaking of translation invariance

→  $p$ -dimensional defect

# Defect operators

## Breaking of translation invariance

- $p$ - dimensional defect
- $\exists$  defect operator  $\Delta^i$  of dimension  $p + 1$

# Defect operators

## Breaking of translation invariance

→  $p$ - dimensional defect

→  $\exists$  defect operator  $\Delta^i$  of dimension  $p + 1$

$$\partial_\mu T^{\mu i}(x) = -D^i(x) \underbrace{\delta^p(x)}_{\text{on defect}}$$

$\perp$  to defect  
 $\downarrow$



# Defect operators

## Breaking of translation invariance

→  $p$ - dimensional defect

→  $\exists$  defect operator  $\Delta^i$  of dimension  $p + 1$

$$\partial_\mu T^{\mu i}(x) = -D^i(x) \underbrace{\delta^p(x)}_{\text{on defect}}$$

$\perp$  to defect  
 $\downarrow$

→ Implements displacements of defect

# Defect operators

## Breaking of translation invariance

→  $p$ - dimensional defect

→  $\exists$  defect operator  $\Delta^i$  of dimension  $p + 1$

$$\partial_\mu T^{\mu i}(x) = -D^i(x) \underbrace{\delta^p(x)}_{\text{on defect}}$$

$\perp$  to defect  
 $\downarrow$

→ Implements displacements of defect

→  $\langle D^i D^j \rangle \propto \delta^{ij} C_D \Rightarrow$  physical meaning

# Defect operators

## Breaking of translation invariance

- $p$ - dimensional defect
- $\exists$  defect operator  $\Delta^i$  of dimension  $p + 1$

$$\partial_\mu T^{\mu i}(x) = -D^i(x) \underbrace{\delta^p(x)}_{\text{on defect}}$$

$\perp$  to defect  
 $\downarrow$

- Implements displacements of defect
- $\langle D^i D^j \rangle \propto \delta^{ij} C_D \Rightarrow$  physical meaning

## Breaking of symmetries by defect

- $\exists$  defect operators associated to that breaking

# Outline

## 1 The bootstrap program

Adding defects

Universality in defect CFT

## 2 Defects in $4d$ $\mathcal{N} = 2$ SCFTs

## 3 Summary & Outlook

# Universality in defect CFT

Defect spectrum looks trivial at large  $s$

[ML Liendo Meineri Sarkar] (analogous to CFT case)

# Universality in defect CFT

Defect spectrum looks trivial at large  $s$

[ML Liendo Meineri Sarkar] (analogous to CFT case)

→  $s \leftarrow SO(d - p)$  spin

# Universality in defect CFT

## Defect spectrum looks trivial at large $s$

[ML Liendo Meineri Sarkar] (analogous to CFT case)

→  $s \leftarrow SO(d - p)$  spin

→ For each bulk scalar  $\phi \Rightarrow \exists$  defect operator with

# Universality in defect CFT

## Defect spectrum looks trivial at large $s$

[ML Liendo Meineri Sarkar] (analogous to CFT case)

→  $s \leftarrow SO(d - p)$  spin

→ For each bulk scalar  $\phi \Rightarrow \exists$  defect operator with

$$\hat{\Delta} \rightarrow \Delta_\phi + 2n + s \text{ as } s \rightarrow \infty$$



# Universality in defect CFT

## Defect spectrum looks trivial at large $s$

[ML Liendo Meineri Sarkar] (analogous to CFT case)

→  $s \leftarrow SO(d - p)$  spin

→ For each bulk scalar  $\phi \Rightarrow \exists$  defect operator with

$$\hat{\Delta} \rightarrow \Delta_\phi + 2n + s \text{ as } s \rightarrow \infty$$

→ trivial defect:

$$\hookrightarrow \underbrace{\partial_\perp^{i_1} \dots \partial_\perp^{i_s}}_{\perp \text{ to defect}} (\partial_\perp \partial_\perp)^n \phi$$

# Universality in defect CFT

## Defect spectrum looks trivial at large $s$

[ML Liendo Meineri Sarkar] (analogous to CFT case)

→  $s \leftarrow SO(d - p)$  spin

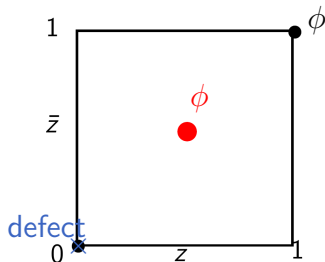
→ For each bulk scalar  $\phi \Rightarrow \exists$  defect operator with

$$\hat{\Delta} \rightarrow \Delta_\phi + 2n + s \text{ as } s \rightarrow \infty$$

→ trivial defect:

$$\hookrightarrow \underbrace{\partial_\perp^{i_1} \dots \partial_\perp^{i_s}}_{\perp \text{ to defect}} (\partial_\perp \partial_\perp)^n \phi$$

$$\langle \phi \phi \rangle$$



# Universality in defect CFT

## Defect spectrum looks trivial at large $s$

[ML Liendo Meineri Sarkar] (analogous to CFT case)

→  $s \leftarrow SO(d-p)$  spin

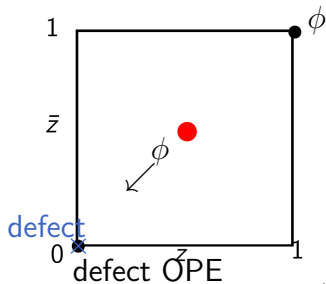
→ For each bulk scalar  $\phi \Rightarrow \exists$  defect operator with

$$\hat{\Delta} \rightarrow \Delta_\phi + 2n + s \text{ as } s \rightarrow \infty$$

→ trivial defect:

$$\hookrightarrow \underbrace{\partial_\perp^{i_1} \dots \partial_\perp^{i_s}}_{\perp \text{ to defect}} (\partial_\perp \partial_\perp)^n \phi$$

$$\langle \phi \phi \rangle = \sum_{\hat{\mathcal{O}}} b_{\phi \hat{\mathcal{O}}}^2 \hat{f}(z, \bar{z})$$



# Universality in defect CFT

## Defect spectrum looks trivial at large $s$

[ML Liendo Meineri Sarkar] (analogous to CFT case)

→  $s \leftarrow SO(d-p)$  spin

→ For each bulk scalar  $\phi \Rightarrow \exists$  defect operator with

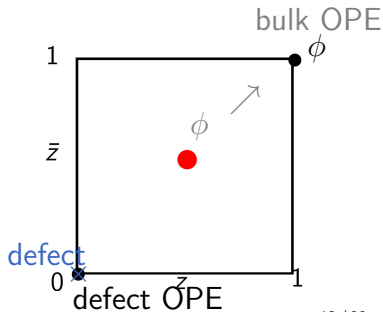
$$\hat{\Delta} \rightarrow \Delta_\phi + 2n + s \text{ as } s \rightarrow \infty$$

→ trivial defect:

$$\hookrightarrow \underbrace{\partial_\perp^{i_1} \dots \partial_\perp^{i_s}}_{\perp \text{ to defect}} (\partial_\perp \partial_\perp)^n \phi$$

$$\langle \phi \phi \rangle = \sum_{\hat{\mathcal{O}}} b_{\phi \hat{\mathcal{O}}}^2 \hat{f}(z, \bar{z})$$

$$= \sum_{\mathcal{O}} f_{\phi \phi \mathcal{O}} a_{\mathcal{O}} g(z, \bar{z})$$



# Universality in defect CFT

## Defect spectrum looks trivial at large $s$

[ML Liendo Meineri Sarkar] (analogous to CFT case)

→  $s \leftarrow SO(d-p)$  spin

→ For each bulk scalar  $\phi \Rightarrow \exists$  defect operator with

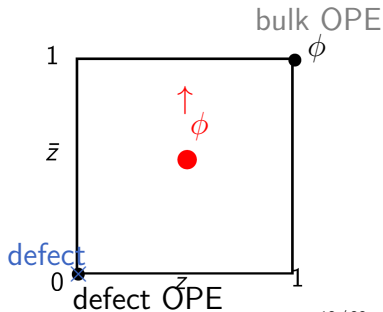
$$\hat{\Delta} \rightarrow \Delta_\phi + 2n + s \text{ as } s \rightarrow \infty$$

→ trivial defect:

$$\hookrightarrow \underbrace{\partial_\perp^{i_1} \dots \partial_\perp^{i_s}}_{\perp \text{ to defect}} (\partial_\perp \partial_\perp)^n \phi$$

$$\langle \phi \phi \rangle = \sum_{\hat{\mathcal{O}}} b_{\phi \hat{\mathcal{O}}}^2 \hat{f}(z, \bar{z})$$

$$= \underbrace{\sum_{\mathcal{O}} f_{\phi \phi \mathcal{O}} a_{\mathcal{O}} g(z, \bar{z})}_{\text{low twist}}$$



# Universality in defect CFT

## Defect spectrum looks trivial at large $s$

[ML Liendo Meineri Sarkar] (analogous to CFT case)

→  $s \leftarrow SO(d-p)$  spin

→ For each bulk scalar  $\phi \Rightarrow \exists$  defect operator with

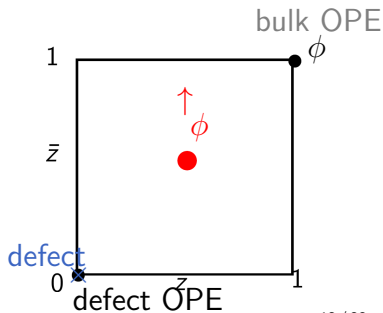
$$\hat{\Delta} \rightarrow \Delta_\phi + 2n + s \text{ as } s \rightarrow \infty$$

→ trivial defect:

$$\hookrightarrow \underbrace{\partial_\perp^{i_1} \dots \partial_\perp^{i_s}}_{\perp \text{ to defect}} (\partial_\perp \partial_\perp)^n \phi$$

$$\langle \phi \phi \rangle = \sum_{\hat{\mathcal{O}}} b_{\phi \hat{\mathcal{O}}}^2 \hat{f}(z, \bar{z}) \longrightarrow \text{large } s$$

$$= \underbrace{\sum_{\mathcal{O}} f_{\phi \phi \mathcal{O}} a_{\mathcal{O}} g(z, \bar{z})}_{\text{low twist}}$$



# Outline

## ① The bootstrap program

Adding defects

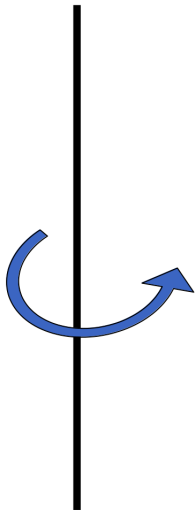
Universality in defect CFT

## ② Defects in $4d$ $\mathcal{N} = 2$ SCFTs

## ③ Summary & Outlook

# Defects in $4d \mathcal{N} = 2$ SCFTs

## Half-BPS lines and surfaces

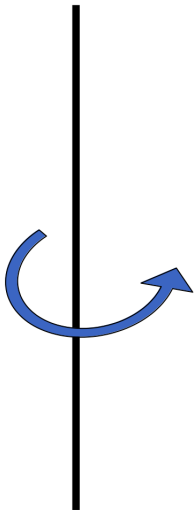




# Defects in $4d \mathcal{N} = 2$ SCFTs

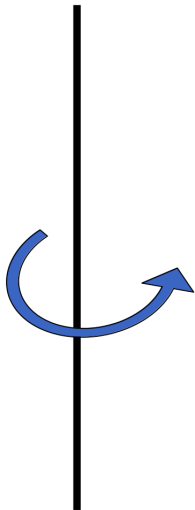
## Half-BPS lines and surfaces

$4d \mathcal{N} = 2$  SCFT



# Defects in $4d \mathcal{N} = 2$ SCFTs

## Half-BPS lines and surfaces

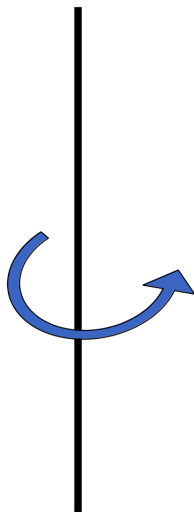


$4d \mathcal{N} = 2$  SCFT

$$su(2, 2|2) \supset \underbrace{so(4, 2)}_{\text{conformal}}$$

# Defects in $4d \mathcal{N} = 2$ SCFTs

## Half-BPS lines and surfaces

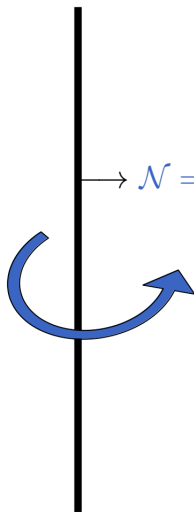


$4d \mathcal{N} = 2$  SCFT

$$su(2, 2|2) \supset \underbrace{so(4, 2)}_{\text{conformal}} \oplus \underbrace{su(2)_R \oplus u(1)_r}_{R\text{-symmetry}}$$

# Defects in $4d \mathcal{N} = 2$ SCFTs

## Half-BPS lines and surfaces



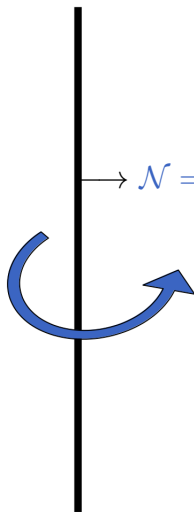
$4d \mathcal{N} = 2$  SCFT

$$su(2, 2|2) \supset \underbrace{so(4, 2)}_{\text{conformal}} \oplus \underbrace{su(2)_R \oplus u(1)_r}_{R\text{-symmetry}}$$

$\rightarrow \mathcal{N} = (2, 2)$  surface

# Defects in $4d \mathcal{N} = 2$ SCFTs

## Half-BPS lines and surfaces



$4d \mathcal{N} = 2$  SCFT

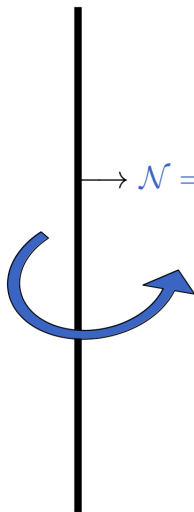
$$su(2, 2|2) \supset \underbrace{so(4, 2)}_{\text{conformal}} \oplus \underbrace{su(2)_R \oplus u(1)_r}_{R\text{-symmetry}}$$

$\rightarrow \mathcal{N} = (2, 2)$  surface

$$su(1, 1|1) \oplus su(1, 1|1)$$

# Defects in $4d \mathcal{N} = 2$ SCFTs

## Half-BPS lines and surfaces



$4d \mathcal{N} = 2$  SCFT

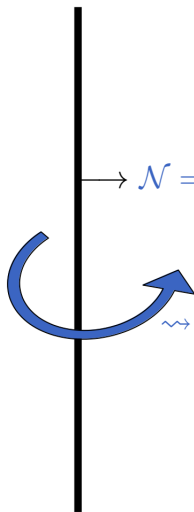
$$su(2, 2|2) \supset \underbrace{so(4, 2)}_{\text{conformal}} \oplus \underbrace{su(2)_R \oplus u(1)_r}_{R\text{-symmetry}}$$

$\rightarrow \mathcal{N} = (2, 2)$  surface

$$su(1, 1|1) \oplus su(1, 1|1) \oplus u(1)_c$$

# Defects in $4d \mathcal{N} = 2$ SCFTs

## Half-BPS lines and surfaces



$4d \mathcal{N} = 2$  SCFT

$$su(2, 2|2) \supset \underbrace{so(4, 2)}_{\text{conformal}} \oplus \underbrace{su(2)_R \oplus u(1)_r}_{R\text{-symmetry}}$$

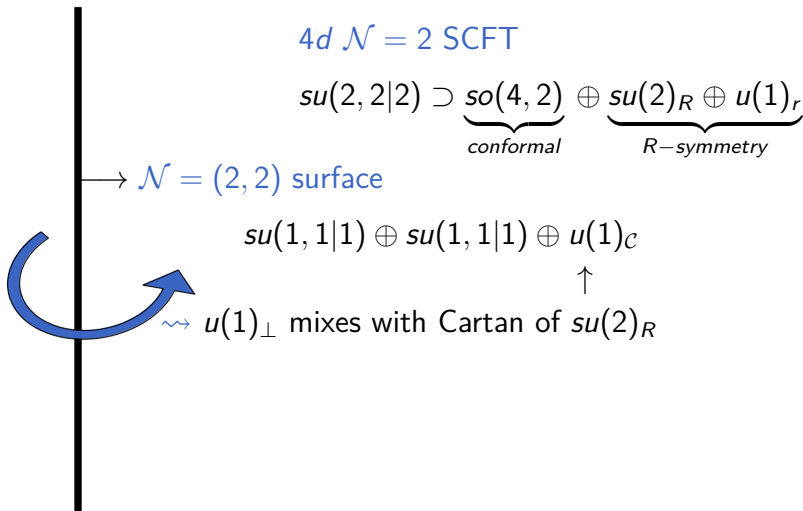
$\rightarrow \mathcal{N} = (2, 2)$  surface

$$su(1, 1|1) \oplus su(1, 1|1) \oplus u(1)_c$$

$\rightsquigarrow u(1)_\perp$

# Defects in $4d \mathcal{N} = 2$ SCFTs

## Half-BPS lines and surfaces





# Displacement operator

## Breaking of translation invariance

$$\rightarrow \langle D^i D^j \rangle \propto \delta^{ij} C_D$$

# Displacement operator

## Breaking of translation invariance

$$\rightarrow \langle D^i D^j \rangle \propto \delta^{ij} C_D$$

$$\rightarrow \langle T^{\mu\nu} \rangle \propto a_T$$

# Displacement operator

## Breaking of translation invariance

$$\rightarrow \langle D^i D^j \rangle \propto \delta^{ij} C_D$$

$$\rightarrow \langle T^{\mu\nu} \rangle \propto a_T$$

**Q:** Relation between  $a_T$  and  $C_D$ ?

# Displacement operator

## Breaking of translation invariance

$$\rightarrow \langle D^i D^j \rangle \propto \delta^{ij} C_D$$

$$\rightarrow \langle T^{\mu\nu} \rangle \propto a_T$$

**Q:** Relation between  $a_T$  and  $C_D$ ?

$\rightarrow$  conjectured relation for [Lewkowycz Maldacena] for certain susy cases

# Displacement operator

## Breaking of translation invariance

$$\rightarrow \langle D^i D^j \rangle \propto \delta^{ij} C_D$$

$$\rightarrow \langle T^{\mu\nu} \rangle \propto a_T$$

**Q:** Relation between  $a_T$  and  $C_D$ ?

→ conjectured relation for [Lewkowycz Maldacena] for certain susy cases

! Unrelated in general

# Displacement operator

## Breaking of translation invariance

$$\rightarrow \langle D^i D^j \rangle \propto \delta^{ij} C_D$$

$$\rightarrow \langle T^{\mu\nu} \rangle \propto a_T$$

**Q:** Relation between  $a_T$  and  $C_D$ ?

→ conjectured relation for [Lewkowycz Maldacena] for certain susy cases

! Unrelated in general

→ When does the relation exist?

# Displacement operator

## Breaking of translation invariance

$$\rightarrow \langle D^i D^j \rangle \propto \delta^{ij} C_D$$

$$\rightarrow \langle T^{\mu\nu} \rangle \propto a_T$$

**Q:** Relation between  $a_T$  and  $C_D$ ?

→ conjectured relation for [Lewkowycz Maldacena] for certain susy cases

! Unrelated in general

→ When does the relation exist?

✓  $\mathcal{N} = 2$  Half-BPS line and surfaces

# Displacement operator

## Breaking of translation invariance

$$\rightarrow \langle D^i D^j \rangle \propto \delta^{ij} C_D$$

$$\rightarrow \langle T^{\mu\nu} \rangle \propto a_T$$

**Q:** Relation between  $a_T$  and  $C_D$ ?

→ conjectured relation for [Lewkowycz Maldacena] for certain susy cases

! Unrelated in general

→ When does the relation exist?

✓  $\mathcal{N} = 2$  Half-BPS line and surfaces

⇒ follows from supersymmetry

[Bianchi ML Meineri, Bianchi ML]



# Broken symmetries

## Displacement supermultiplet

Broken translations  $\Rightarrow$

# Broken symmetries

## Displacement supermultiplet

Broken translations  $\Rightarrow$  Displacement

# Broken symmetries

## Displacement supermultiplet

Broken  $su(2)_R \rightarrow u(1) \Rightarrow$

Broken translations  $\Rightarrow$  Displacement

# Broken symmetries

## Displacement supermultiplet

Broken  $su(2)_R \rightarrow u(1) \quad \Rightarrow \quad \textcircled{0}$

Broken translations  $\Rightarrow$  Displacement

# Broken symmetries

## Displacement supermultiplet

Broken  $su(2)_R \rightarrow u(1) \Rightarrow \textcircled{0}$

Broken supersymmetry  $Q, \tilde{Q} \Rightarrow$

Broken translations  $\Rightarrow$  Displacement

# Broken symmetries

## Displacement supermultiplet

Broken  $su(2)_R \rightarrow u(1)$   $\Rightarrow$   $\mathbb{O}$

Broken supersymmetry  $Q, \tilde{Q}$   $\Rightarrow$   $\Lambda$   $\tilde{\Lambda}$

Broken translations  $\Rightarrow$  Displacement

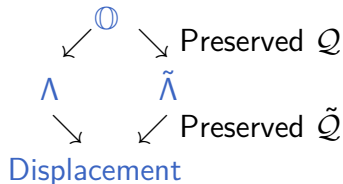
# Broken symmetries

## Displacement supermultiplet

Broken  $su(2)_R \rightarrow u(1)$   $\Rightarrow$

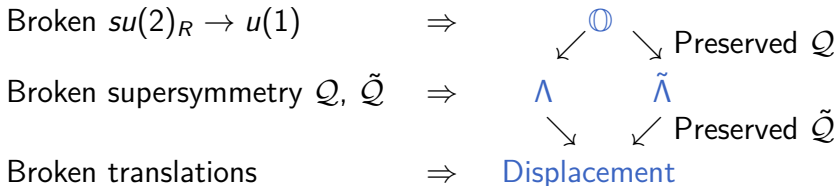
Broken supersymmetry  $Q, \tilde{Q}$   $\Rightarrow$

Broken translations  $\Rightarrow$



# Broken symmetries

## Displacement supermultiplet



→ *Preserved* supersymmetry relates two-point functions



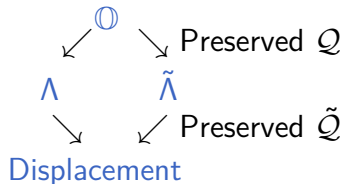
# Broken symmetries

## Displacement supermultiplet

Broken  $su(2)_R \rightarrow u(1) \Rightarrow$

Broken supersymmetry  $Q, \tilde{Q} \Rightarrow$

Broken translations  $\Rightarrow$



$\rightarrow$  *Preserved* supersymmetry relates two-point functions

## Stress tensor supermultiplet

$\rightarrow \mathcal{O}_2, \dots, J_{su(2)_R}^\mu, J_{u(1)_r}^\mu, \dots, T^{\mu\nu}$

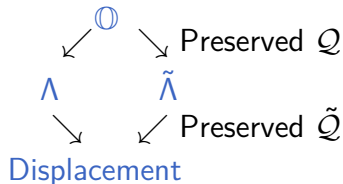
# Broken symmetries

## Displacement supermultiplet

Broken  $su(2)_R \rightarrow u(1)$   $\Rightarrow$

Broken supersymmetry  $Q, \tilde{Q}$   $\Rightarrow$

Broken translations  $\Rightarrow$



$\rightarrow$  *Preserved* supersymmetry relates two-point functions

## Stress tensor supermultiplet

$\rightarrow \mathcal{O}_2, \dots, J_{su(2)_R}^\mu, J_{u(1)_r}^\mu, \dots, T^{\mu\nu}$

$\rightarrow$  *Preserved* supersymmetry relates one-point functions

# Stress tensor - displacement coupling

$$\langle T^{\mu\nu} D^i \rangle$$

# Stress tensor - displacement coupling

$$\langle T^{\mu\nu} D^i \rangle$$

→ Ward identities for *broken* translations

$$\rightarrow \langle T_{\mu\nu} D^i \rangle \propto C_D, a_T$$

[Billò Gonçalves Lauria Meineri]

# Stress tensor - displacement coupling

$$\langle T^{\mu\nu} D^i \rangle$$

- Ward identities for *broken* translations
- $\langle T_{\mu\nu} D^i \rangle \propto C_D, a_T$

[Billò Gonçalves Lauria Meineri]

## Super stress tensor - super displacement coupling

- Preserved  $Q, \tilde{Q} \Rightarrow$  relate two-point functions

# Stress tensor - displacement coupling

$$\langle T^{\mu\nu} D^i \rangle$$

- Ward identities for *broken* translations
- $\langle T_{\mu\nu} D^i \rangle \propto C_D, a_T$

[Billò Gonçalves Lauria Meineri]

## Super stress tensor - super displacement coupling

- Preserved  $Q, \tilde{Q} \Rightarrow$  relate two-point functions
- Broken symmetries  $\Rightarrow$  Ward identities

# Stress tensor - displacement coupling

$$\langle T^{\mu\nu} D^i \rangle$$

- Ward identities for *broken* translations
- $\langle T_{\mu\nu} D^i \rangle \propto C_D, a_T$

[Billò Gonçalves Lauria Meineri]

## Super stress tensor - super displacement coupling

- Preserved  $Q, \tilde{Q} \Rightarrow$  relate two-point functions
- Broken symmetries  $\Rightarrow$  Ward identities
- $\Rightarrow C_D = -12a_T$

# A solvable subsector of $4d \mathcal{N} \geq 2$ SCFTs

$4d \mathcal{N} = 2$  SCFTs  $\rightarrow$   $2d$  chiral algebra

[Beem Lemos Liendo Peelaers Rastelli van Rees] [see Pufu's talk]



# A solvable subsector of $4d \mathcal{N} \geq 2$ SCFTs

$4d \mathcal{N} = 2$  SCFTs  $\rightarrow$   $2d$  chiral algebra

[Beem Lemos Liendo Peelaers Rastelli van Rees] [see Pufu's talk]

$\rightarrow$  Restrict operators to plane

# A solvable subsector of $4d \mathcal{N} \geq 2$ SCFTs

$4d \mathcal{N} = 2$  SCFTs  $\rightarrow$   $2d$  chiral algebra

[Beem Lemos Liendo Peelaers Rastelli van Rees] [see Pufu's talk]

$\rightarrow$  Restrict operators to plane

$\rightarrow$  Cohomology of  $\mathcal{Q} + \mathcal{S} \Rightarrow 2d$  chiral algebra

# A solvable subsector of $4d \mathcal{N} \geq 2$ SCFTs

$4d \mathcal{N} = 2$  SCFTs  $\rightarrow$   $2d$  chiral algebra

[Beem Lemos Liendo Peelaers Rastelli van Rees] [see Pufu's talk]

$\rightarrow$  Restrict operators to plane

$\rightarrow$  Cohomology of  $\mathcal{Q} + \mathcal{S} \Rightarrow 2d$  chiral algebra

# A solvable subsector of $4d \mathcal{N} \geq 2$ SCFTs

$4d \mathcal{N} = 2$  SCFTs  $\rightarrow$   $2d$  chiral algebra

[Beem Lemos Liendo Peelaers Rastelli van Rees] [see Pufu's talk]

- $\rightarrow$  Restrict operators to plane
- $\rightarrow$  Cohomology of  $\mathcal{Q} + \mathcal{S} \Rightarrow 2d$  chiral algebra
- $\rightarrow su(2)_R$  current  $\mapsto 2d$  stress tensor  $T(z)$

# A solvable subsector of $4d \mathcal{N} \geq 2$ SCFTs

$4d \mathcal{N} = 2$  SCFTs  $\rightarrow$   $2d$  chiral algebra

[Beem Lemos Liendo Peelaers Rastelli van Rees] [see Pufu's talk]

$\rightarrow$  Restrict operators to plane

$\rightarrow$  Cohomology of  $\mathcal{Q} + \mathcal{S} \Rightarrow 2d$  chiral algebra

$\rightarrow$   $\underbrace{su(2)_R}$  current  $\mapsto 2d$  stress tensor  $T(z)$

$\in$  Super-stress tensor multiplet

# A solvable subsector of $4d \mathcal{N} \geq 2$ SCFTs

$4d \mathcal{N} = 2$  SCFTs  $\rightarrow$   $2d$  chiral algebra

[Beem Lemos Liendo Peelaers Rastelli van Rees] [see Pufu's talk]

$\rightarrow$  Restrict operators to plane

$\rightarrow$  Cohomology of  $\mathcal{Q} + \mathcal{S} \Rightarrow 2d$  chiral algebra

$\rightarrow$   $\underbrace{su(2)_R}$  current  $\mapsto 2d$  stress tensor  $T(z)$

$\in$  Super-stress tensor multiplet

$$\hookrightarrow J_{su(2)_R}^\mu(z, \bar{z}) J_{su(2)_R}^\nu(z, \bar{z}) \sim \dots$$

# A solvable subsector of $4d \mathcal{N} \geq 2$ SCFTs

$4d \mathcal{N} = 2$  SCFTs  $\rightarrow$   $2d$  chiral algebra

[Beem Lemos Liendo Peelaers Rastelli van Rees] [see Pufu's talk]

$\rightarrow$  Restrict operators to plane

$\rightarrow$  Cohomology of  $\mathcal{Q} + \mathcal{S} \Rightarrow 2d$  chiral algebra

$\rightarrow$   $\underbrace{su(2)_R}$  current  $\mapsto 2d$  stress tensor  $T(z)$

$\in$  Super-stress tensor multiplet

$$\hookrightarrow J_{su(2)_R}^\mu(z, \bar{z}) J_{su(2)_R}^\nu(z, \bar{z}) \sim \dots$$

$$\hookrightarrow T(z) T(0) \sim -12 \frac{c_{4d}/2}{z^4} + 2 \frac{T(0)}{z^2} + \frac{\partial T(0)}{z} + \dots$$

# A solvable subsector of $4d \mathcal{N} \geq 2$ SCFTs

$4d \mathcal{N} = 2$  SCFTs  $\rightarrow$   $2d$  chiral algebra

[Beem Lemos Liendo Peelaers Rastelli van Rees] [see Pufu's talk]

- $\rightarrow$  Restrict operators to plane
- $\rightarrow$  Cohomology of  $\mathcal{Q} + \mathcal{S} \Rightarrow 2d$  chiral algebra
- $\rightarrow$   $\underbrace{su(2)_R}$  current  $\mapsto 2d$  stress tensor  $T(z)$

$\in$  Super-stress tensor multiplet

$$\hookrightarrow J_{su(2)_R}^\mu(z, \bar{z}) J_{su(2)_R}^\nu(z, \bar{z}) \sim \dots$$

$$\hookrightarrow T(z) T(0) \sim -12 \frac{c_{4d}/2}{z^4} + 2 \frac{T(0)}{z^2} + \frac{\partial T(0)}{z} + \dots$$

$$\hookrightarrow \text{Global } sl(2) \text{ enhances to Virasoro}$$



# A solvable subsector of $4d \mathcal{N} \geq 2$ SCFTs

$4d \mathcal{N} = 2$  SCFTs  $\rightarrow$   $2d$  chiral algebra

[Beem Lemos Liendo Peelaers Rastelli van Rees] [see Pufu's talk]

- $\rightarrow$  Restrict operators to plane
- $\rightarrow$  Cohomology of  $\mathcal{Q} + \mathcal{S} \Rightarrow 2d$  chiral algebra
- $\rightarrow$   $\underbrace{su(2)_R}$  current  $\mapsto 2d$  stress tensor  $T(z)$

$\in$  Super-stress tensor multiplet

$$\hookrightarrow J_{su(2)_R}^\mu(z, \bar{z}) J_{su(2)_R}^\nu(z, \bar{z}) \sim \dots$$

$$\hookrightarrow T(z) T(0) \sim -12 \frac{c_{4d}/2}{z^4} + 2 \frac{T(0)}{z^2} + \frac{\partial T(0)}{z} + \dots$$

$\hookrightarrow$  Global  $sl(2)$  enhances to Virasoro

$$\hookrightarrow c_{2d} = -12c_{4d}$$

# A solvable subsector of $4d \mathcal{N} \geq 2$ SCFTs

$4d \mathcal{N} = 2$  SCFTs  $\rightarrow$   $2d$  chiral algebra

[Beem Lemos Liendo Peelaers Rastelli van Rees] [see Pufu's talk]

- $\rightarrow$  Restrict operators to plane
- $\rightarrow$  Cohomology of  $\mathcal{Q} + \mathcal{S} \Rightarrow 2d$  chiral algebra
- $\rightarrow$   $\underbrace{su(2)}_R$  current  $\mapsto 2d$  stress tensor  $T(z)$

$\in$  Super-stress tensor multiplet

$$\hookrightarrow J_{su(2)_R}^\mu(z, \bar{z}) J_{su(2)_R}^\nu(z, \bar{z}) \sim \dots$$

$$\hookrightarrow T(z) T(0) \sim -12 \frac{c_{4d}/2}{z^4} + 2 \frac{T(0)}{z^2} + \frac{\partial T(0)}{z} + \dots$$

$\hookrightarrow$  Global  $sl(2)$  enhances to Virasoro

$$\hookrightarrow \boxed{c_{2d} = -12c_{4d}}$$

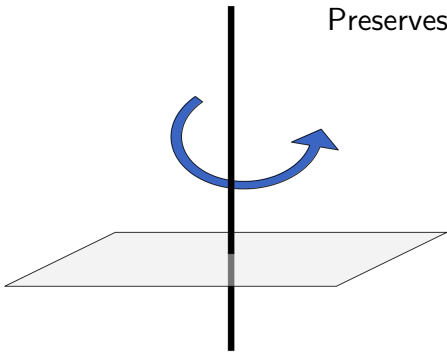
- $\rightarrow$  Each  $\mathcal{N} = 2$  multiplet contributes at most with one  $sl(2)$  primary

# Adding defects

## Surfaces preserving $\mathcal{N} = (2, 2)$

[Beem Peelaers Rastelli, Cordova Gaiotto Shao]

Preserves  $\mathcal{Q} + \mathcal{S}$  used for chiral algebra

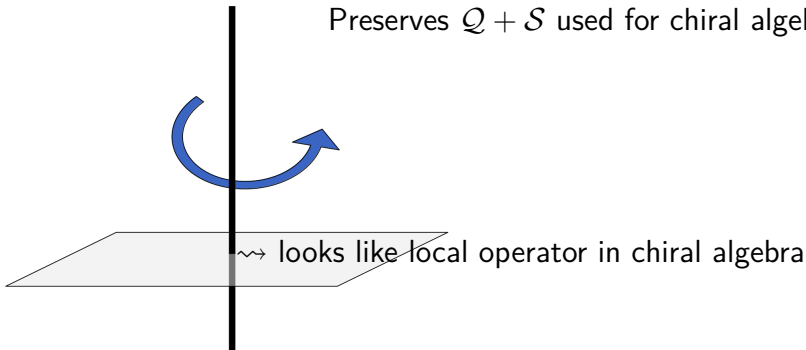


# Adding defects

## Surfaces preserving $\mathcal{N} = (2, 2)$

[Beem Peelaers Rastelli, Cordova Gaiotto Shao]

Preserves  $\mathcal{Q} + \mathcal{S}$  used for chiral algebra

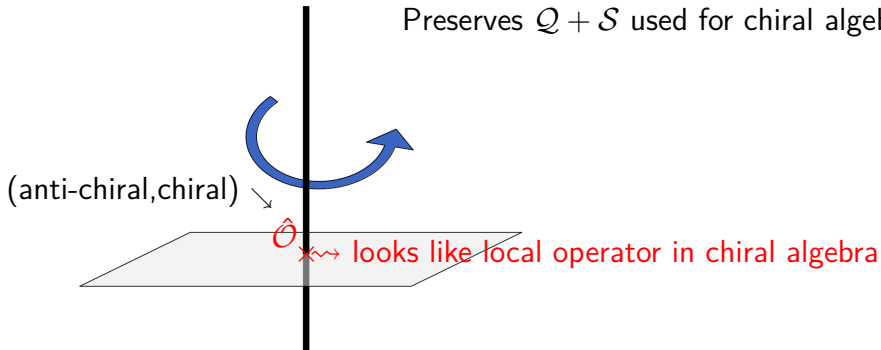


# Adding defects

## Surfaces preserving $\mathcal{N} = (2, 2)$

[Beem Peelaers Rastelli, Cordova Gaiotto Shao]

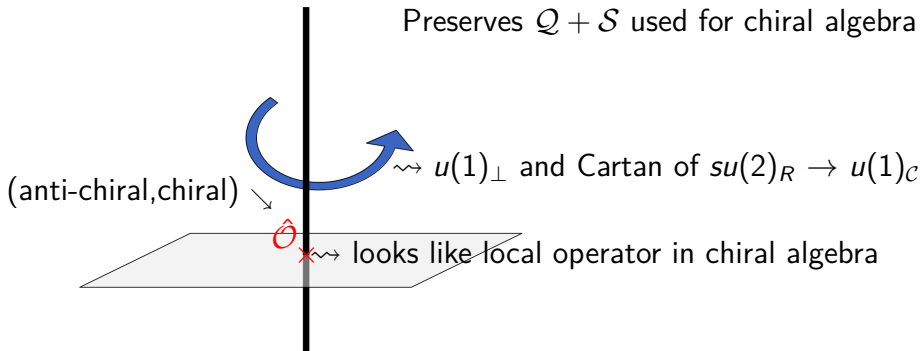
Preserves  $\mathcal{Q} + \mathcal{S}$  used for chiral algebra



# Adding defects

## Surfaces preserving $\mathcal{N} = (2, 2)$

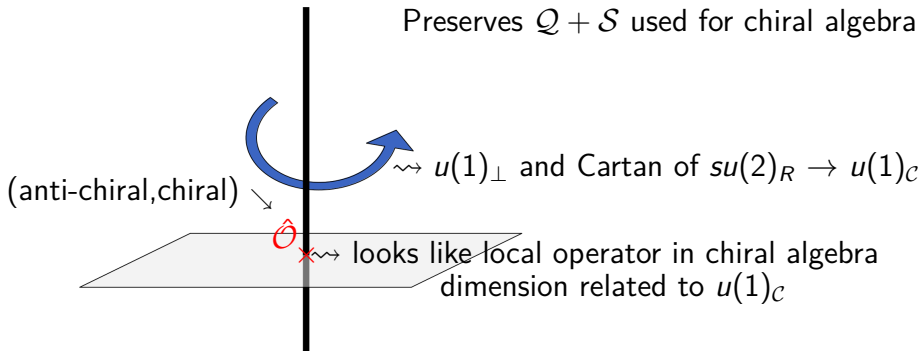
[Beem Peelaers Rastelli, Cordova Gaiotto Shao]



# Adding defects

## Surfaces preserving $\mathcal{N} = (2, 2)$

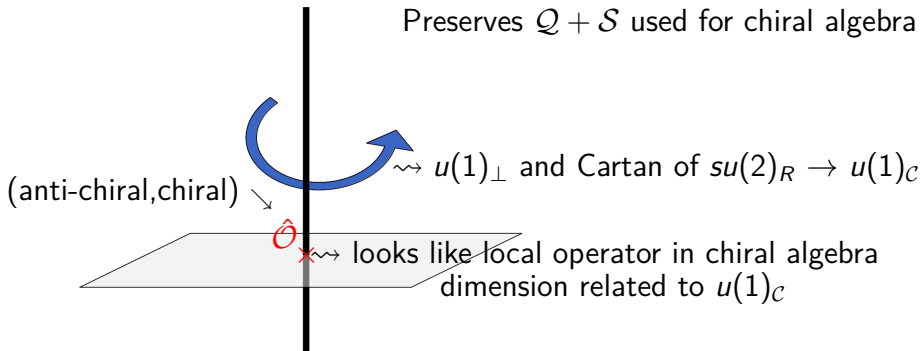
[Beem Peelaers Rastelli, Cordova Gaiotto Shao]



# Adding defects

## Surfaces preserving $\mathcal{N} = (2, 2)$

[Beem Peelaers Rastelli, Cordova Gaiotto Shao]



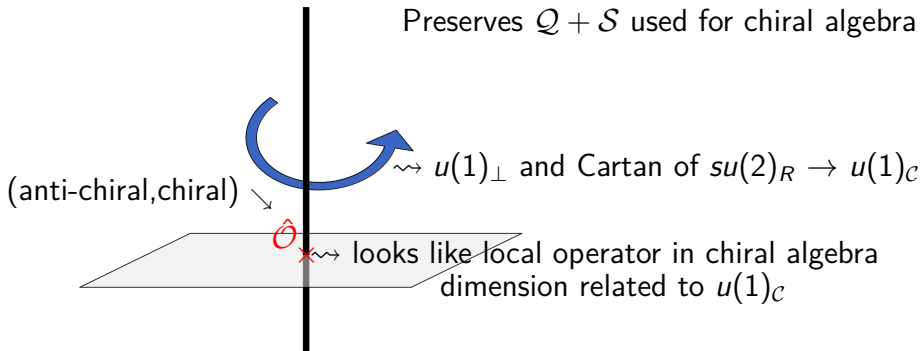
→ Defects give modules of chiral algebra



# Adding defects

## Surfaces preserving $\mathcal{N} = (2, 2)$

[Beem Peelaers Rastelli, Cordova Gaiotto Shao]



- Defects give modules of chiral algebra
- ↪ Schur indices matched to characters of modules

# Chiral algebras with defects

## Operators in cohomology

→ (anti-chiral, chiral) defect operators

# Chiral algebras with defects

## Operators in cohomology

- (anti-chiral, chiral) defect operators
- Defect identity  $\mapsto \sigma$

# Chiral algebras with defects

## Operators in cohomology

- (anti-chiral, chiral) defect operators
- Defect identity  $\mapsto \sigma$ 
  - ↪  $J_{su(2)_R}^\mu$  defect OPE selection rules

# Chiral algebras with defects

## Operators in cohomology

- (anti-chiral, chiral) defect operators
- Defect identity  $\mapsto \sigma$ 
  - ↪  $J_{su(2)_R}^\mu$  defect OPE selection rules
  - ↪  $T(z)\sigma(0) \sim \frac{h_\sigma \sigma(0) + \dots}{z^2} + \frac{\partial \sigma}{z}$

# Chiral algebras with defects

## Operators in cohomology

- (anti-chiral, chiral) defect operators
- Defect identity  $\mapsto \sigma$ 
  - ↪  $J_{su(2)_R}^\mu$  defect OPE selection rules
  - ↪  $T(z)\sigma(0) \sim \frac{h_\sigma \sigma(0) + \dots}{z^2} + \frac{\partial \sigma}{z}$
- ⇒  $L_{+n}|\sigma\rangle = 0, n > 0$

# Chiral algebras with defects

## Operators in cohomology

- (anti-chiral, chiral) defect operators
- Defect identity  $\mapsto \sigma$ 
  - ↪  $J_{su(2)_R}^\mu$  defect OPE selection rules
  - ↪  $T(z)\sigma(0) \sim \frac{h_\sigma \sigma(0) + \dots}{z^2} + \frac{\partial \sigma}{z}$
- ⇒  $L_{+n}|\sigma\rangle = 0, n > 0$
- ⇒  $h_\sigma \propto a_T$

# Chiral algebras with defects

## Operators in cohomology

- (anti-chiral, chiral) defect operators
- Defect identity  $\mapsto \sigma$ 
  - ↪  $J_{su(2)_R}^\mu$  defect OPE selection rules
  - ↪  $T(z)\sigma(0) \sim \frac{h_\sigma \sigma(0) + \dots}{z^2} + \frac{\partial \sigma}{z}$
- ⇒  $L_{+n}|\sigma\rangle = 0, n > 0$
- ⇒  $h_\sigma \propto a_{\mathcal{T}}$
- ⇒  $L_0$  may not act diagonally



# Chiral algebras with defects

## Operators in cohomology

- (anti-chiral, chiral) defect operators
- Defect identity  $\mapsto \sigma$ 
  - ↪  $J_{su(2)_R}^\mu$  defect OPE selection rules
  - ↪  $T(z)\sigma(0) \sim \frac{h_\sigma \sigma(0) + \dots}{z^2} + \frac{\partial \sigma}{z}$
- ⇒  $L_{+n}|\sigma\rangle = 0, n > 0$
- ⇒  $h_\sigma \propto a_T$
- ⇒  $L_0$  may not act diagonally
- Superprimary of displacement

# Chiral algebras with defects

## Operators in cohomology

- (anti-chiral, chiral) defect operators
- Defect identity  $\mapsto \sigma$ 
  - ↪  $J_{su(2)_R}^\mu$  defect OPE selection rules
  - ↪  $T(z)\sigma(0) \sim \frac{h_\sigma \sigma(0) + \dots}{z^2} + \frac{\partial \sigma}{z}$
- ⇒  $L_{+n}|\sigma\rangle = 0, n > 0$
- ⇒  $h_\sigma \propto a_T$
- ⇒  $L_0$  may not act diagonally
- Superprimary of displacement
  - ↪ dimension  $h_\sigma + 1$

# Chiral algebras with defects

## Operators in cohomology

- (anti-chiral, chiral) defect operators
- Defect identity  $\mapsto \sigma$ 
  - ↪  $J_{su(2)_R}^\mu$  defect OPE selection rules
  - ↪  $T(z)\sigma(0) \sim \frac{h_\sigma \sigma(0) + \dots}{z^2} + \frac{\partial \sigma}{z}$
- ⇒  $L_{+n}|\sigma\rangle = 0, n > 0$
- ⇒  $h_\sigma \propto a_T$
- ⇒  $L_0$  may not act diagonally
- Superprimary of displacement
  - ↪ dimension  $h_\sigma + 1$
- Defect marginal operators

# Outline

## ① The bootstrap program

Adding defects

Universality in defect CFT

## ② Defects in $4d$ $\mathcal{N} = 2$ SCFTs

## ③ Summary & Outlook

# Summary & Outlook

Defects in conformal field theories

# Summary & Outlook

## Defects in conformal field theories

- ▶ Universality in defect spectrum

# Summary & Outlook

## Defects in conformal field theories

- ▶ Universality in defect spectrum
- ▶ Half-BPS surfaces and lines in  $4d \mathcal{N} = 2$ :

# Summary & Outlook

## Defects in conformal field theories

- ▶ Universality in defect spectrum
- ▶ Half-BPS surfaces and lines in  $4d$   $\mathcal{N} = 2$ :
- ▶ Supersymmetry  $\Rightarrow$  relation between  $\langle T^{\mu\nu} \rangle$  and  $\langle D^i D^j \rangle$



# Summary & Outlook

## Defects in conformal field theories

- ▶ Universality in defect spectrum
- ▶ Half-BPS surfaces and lines in  $4d \mathcal{N} = 2$ :
- ▶ Supersymmetry  $\Rightarrow$  relation between  $\langle T^{\mu\nu} \rangle$  and  $\langle D^i D^j \rangle$
- ▶ Also in half-BPS surfaces in  $4d \mathcal{N} = 1$ ?

# Summary & Outlook

## Defects in conformal field theories

- ▶ Universality in defect spectrum
- ▶ Half-BPS surfaces and lines in  $4d \mathcal{N} = 2$ :
- ▶ Supersymmetry  $\Rightarrow$  relation between  $\langle T^{\mu\nu} \rangle$  and  $\langle D^i D^j \rangle$
- ▶ Also in half-BPS surfaces in  $4d \mathcal{N} = 1$ ?
- ▶ Generic properties of chiral algebras of defects

# Summary & Outlook

## Defects in conformal field theories

- ▶ Universality in defect spectrum
- ▶ Half-BPS surfaces and lines in  $4d$   $\mathcal{N} = 2$ :
- ▶ Supersymmetry  $\Rightarrow$  relation between  $\langle T^{\mu\nu} \rangle$  and  $\langle D^i D^j \rangle$
- ▶ Also in half-BPS surfaces in  $4d$   $\mathcal{N} = 1$ ?
- ▶ Generic properties of chiral algebras of defects
- ▶ Defect CFT data from chiral algebras?

**Thank you!**